reflection coefficient can be computed from

$$\Gamma = \frac{Z_{11} - 1}{Z_{11} + 1} \tag{36}$$

or, combining with (5),

$$\Gamma = \frac{(\beta - 1) + j2Q_0\delta(\alpha - 1)}{(\beta + 1) + j2Q_0\delta(\alpha + 1)}. \tag{37}$$

Thus, when $2Q_0\delta = \pm\infty$,

$$\overline{DF} = |\Gamma_\infty|$$

$$\overline{DF} = \frac{1 - \alpha}{1 + \alpha}. \tag{38}$$

The distance

$$\overline{DG} = \overline{DF} + \Gamma$$

$$\overline{DG} = \frac{1 - \alpha}{1 + \alpha} + \frac{(\beta - 1) + j2Q_0\delta(\alpha - 1)}{(\beta + 1) + j2Q_0\delta(\alpha + 1)} \tag{39}$$

$$= \frac{2(\beta - \alpha)}{(\alpha + 1)(\beta + 1) + j(\alpha + 1)^2 2Q_0\delta}. \tag{40}$$

The phase angle Φ, shown in Fig. 7, is equal to argument $\overline{DG}$ and is found from the ratio of imaginary to real parts of (40):

$$\Phi = \tan^{-1} \frac{2Q_0\delta(\alpha + 1)}{\beta + 1}. \tag{41}$$

The distance $\overline{FH}$, being proportional to tan Φ, is proportional to δ, *i.e.*, the frequency. Thus, the points projected from the locus onto the straight line A-B produce intercepts whose lengths are proportional to frequency (*Q.E.D.*).

The Excitation of a Dielectric Rod by a Cylindrical Waveguide*

C. M. ANGULO† AND W. S. C. CHANG‡

Summary—**This paper is a theoretical analysis of the excitation of the lowest circular symmetric TM surface wave along an infinite circular dielectric rod by a metallic cylindrical waveguide coaxial with the rod. The asymptotic expressions for all the fields are obtained by means of the Wiener-Hopf method. The expressions for the total average power transmitted to the surface wave, the total average power reflected, and the total power radiated, per unit incident power, are derived and computed for $\epsilon = 2.49$ for various radii of the dielectric rod.**

Introduction

It is well known that a TM circular symmetric surface wave can be easily launched along a circular dielectric rod by a metallic cylindrical waveguide. A condensed theoretical analysis of an idealized version of this problem is given here. For the detailed analysis, the reader is referred to a previous report by the authors.[1]

The structure under consideration is represented in Fig. 1. It consists of an infinite circular dielectric rod of relative permittivity ϵ and radius a fitted tight into a semi-infinite cylindrical waveguide of infinitely thin metallic wall which extends from $z = -\infty$ to $z = 0$.

The incident energy is carried by the $TM_{0,1}$ mode of the cylindrical metallic waveguide. It excites a TM surface wave along the rod, a reflected wave in the waveguide, and a scattered radiation at the end of the metallic waveguide. It is assumed here that along the dielectric rod only the lowest circular symmetric surface wave can exist and that the $TM_{0,1}$ mode is the only mode propagating inside the waveguide. This is true if $2.405(\epsilon - 1)^{-1/2} < Ka < 5.52\epsilon^{-1/2}$, where $K = 2\pi/\lambda_0$.

Since the structure considered (see Fig. 1) is independent of ϕ and the incident wave is the $TM_{0,1}$ mode, only the circular symmetric TM proper and improper modes are excited. Therefore, $\partial/\partial\phi = 0$ and $H_\rho = E_\phi = H_z = 0$. Furthermore, all the higher TM modes excited inside the cylindrical guide attenuate exponentially in the negative z direction. It follows immediately that the far zone fields of our problem must be of the forms:

$$E_z = AJ_0(K_c\rho) \exp\left[-j(K^2\epsilon - K_c^2)^{1/2}z\right] + BJ_0(K_c\rho) \exp\left[j(K^2\epsilon - K_c^2)^{1/2}z\right] \tag{1}$$

$$H_\phi = \frac{jA\omega\epsilon\epsilon_0}{K_c} J_1(K_c\rho) \exp\left[-j(K^2\epsilon - K_c^2)^{1/2}z\right] + j\frac{B\omega\epsilon\epsilon_0}{K_c} J_1(K_c\rho) \exp\left[j(K^2\epsilon - K_c^2)^{1/2}z\right] \tag{2}$$

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† Brown University, Providence, R. I.
‡ Electronics Res. Lab., Stanford University, Stanford, Calif. Formerly at Brown University, Providence, R. I.

[1] C. M. Angulo and W. S. Chang, "Excitation of a Dielectric Rod by a Cylindrical Waveguide," Div. of Eng., Brown University, Providence, R. I., Scientific Rep. AF 1391/7; July, 1957.

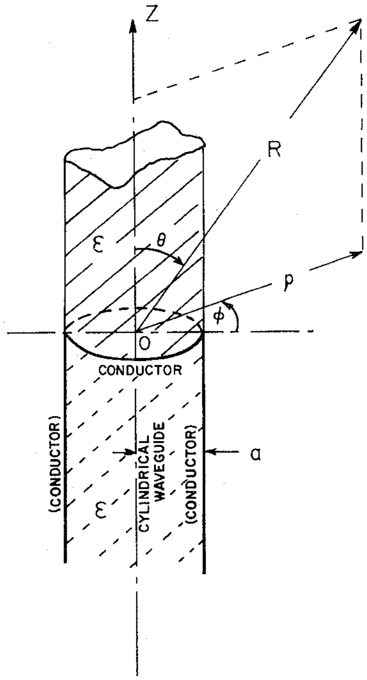

Fig. 1—The dielectric rod excited by a cylindrical waveguide.

for $Kz \ll -1$, inside the metallic waveguide

$$E_z = \frac{-g(\theta)\sin\theta e^{-jKR}}{R} + C\left[K_0(K\rho q)U(\rho-a) + \frac{K_0(Kaq)J_0(K\rho p)}{J_0(Kap)}U(a-\rho)\right] \cdot U(z)\exp\left[-jK(1+q^2)^{1/2}z\right] \quad (3)$$

$$H_\phi = \frac{\epsilon_0^{1/2}g(\theta)e^{-jKR}}{\mu_0^{1/2}R} - j\frac{\epsilon_0^{1/2}C}{\mu_0^{1/2}q}\left[K_1(K\rho q)U(\rho-a) + \frac{K_1(Kaq)J_1(K\rho p)}{J_1(Kap)} \cdot U(a-\rho)\right]U(z)\exp\left[-jK(1+q^2)^{1/2}z\right] \quad (4)$$

for $KR \gg 1$, outside the metallic waveguide, where U stands for the Heaviside unit step function [$U(x)=0$ for $x<0$; $U(x)=1$ for $x>0$]. A is the amplitude of the incident $TM_{0,1}$ mode inside the metallic cylindrical waveguide. B is the amplitude of the reflected $TM_{0,1}$ mode inside the metallic waveguide. C is the amplitude of the transmitted surface wave along the dielectric rod. $g(\theta)$ is the angular distribution function of the radiation field. K_c is the cutoff wave number of the $TM_{0,1}$ mode inside the metallic guide ($K_c a$ = first zero of J_0).

p and q are the propagation wave numbers of the lowest TM circular symmetric surface wave.[2]

$$p^2 + q^2 = \epsilon - 1 \quad (5)$$

$$\epsilon q J_1(Kpa)K_0(Kqa) + pJ_0(Kpa)K_1(Kqa) = 0. \quad (6)$$

[2] S. A. Schelkunoff, "Electro-magnetic Waves," D. Van Nostrand Co., Inc., New York, N. Y., pp. 425–428; 1943.

The Representation of the Fields

Mathematically, we separate the total fields into two parts,

$$E_z = E_{0z} + \mathcal{E}_z, \qquad H_\phi = H_{0\phi} + \mathcal{H}_\phi, \quad (7)$$

where

$$H_{0\phi} = A\frac{j\omega\epsilon_0\epsilon}{K_c}J_1(K_c\rho)U(a-\rho)\exp\left[-j(K^2\epsilon - K_c^2)z\right] \quad (8)$$

$$E_{0z} = AJ_0(K_c\rho)U(a-\rho)\exp\left[-j(K^2\epsilon - K_c^2)z\right]. \quad (9)$$

$H_{0\phi}$ and E_{0z} obviously have the meaning of the incident wave. Whereas, $\mathcal{H}_\phi$ and $\mathcal{E}_z$ are the incomplete scattered fields. We represent these two incomplete scattered fields $\mathcal{H}_\phi$ and $\mathcal{E}_z$ by their Fourier transforms,

$$\mathcal{H}_\phi = \frac{1}{(2\pi)^{1/2}}\int_{-\infty}^{+\infty}\left[\frac{I(\eta,\rho)}{\rho}\right]e^{-j\eta z}d\eta, \quad (10)$$

$$\mathcal{E}_z = \frac{-1}{(2\pi)^{1/2}}\int_{-\infty}^{+\infty}\left[V(\eta,\rho)\right]e^{-j\eta z}d\eta. \quad (11)$$

The Equation of the Fourier Transforms at $\rho = a$

Since E_{0z} and $H_{0\phi}$ satisfy the steady-state Maxwell's equations with time dependence $e^{+j\omega t}$ for $\rho \lessgtr a$, $\mathcal{E}_z$ and $\mathcal{H}_\phi$ must satisfy them also in the same regions. This means that $V(\eta,\rho)$ and $I(\eta,\rho)$ for $\rho \lessgtr a$ are[1]

$$V = V(\eta, a_-)\frac{J_0(\Lambda_d\rho)}{J_0(\Lambda_d a)}U(a_- - \rho) + V(\eta, a_+)\frac{H_0^{(2)}(\Lambda_a\rho)}{H_0^{(2)}(\Lambda_a a)}U(\rho - a_+), \quad (12)$$

$$I = -jY_dV(\eta, a_-)\frac{J_1(\Lambda_d\rho)}{J_0(\Lambda_d a)}U(a_- - \rho) - jY_aV(\eta, a_+)\frac{H_1^{(2)}(\Lambda_a\rho)}{H_0^{(2)}(\Lambda_a a)}U(\rho - a_+), \quad (13)$$

$$\Lambda_a = (K^2 - \eta^2)^{1/2}, \quad (14a)$$

$$\Lambda_d = (K^2\epsilon - \eta^2)^{1/2} \quad (14b)$$

$$Z_a = \frac{1}{Y_a} = \frac{(K^2 - \eta^2)^{1/2}}{\omega\epsilon_0\rho}, \quad (15a)$$

$$Z_d = \frac{1}{Y_d} = \frac{(K^2\epsilon - \eta^2)^{1/2}}{\omega\epsilon_0\epsilon\rho} \quad (15b)$$

where the subscript plus means $\lim_{\rho\to a}$ from $\rho > a$ and the subscript minus means $\lim_{\rho\to a}$ from $\rho < a$.

At $\rho = a$, $V(\eta,\rho)$ and $I(\eta,\rho)$ must satisfy the boundary conditions determined by E_z and H_ϕ, *i.e.*,

1) $\mathcal{E}_z = 0$ for $z<0$.
2) $\mathcal{E}_z(z, a_+) = \mathcal{E}_z(z, a_-) = \mathcal{E}_z(z, a)$ for all z.
3) $\mathcal{H}_\phi(z, a_+) - \mathcal{H}_\phi(z, a_-) = H_{0\phi}(z, a_-)$ for $z>0$.

Let

$$\mathcal{G}^+(\eta, a) = \frac{1}{(2\pi)^{1/2}}\int_0^{+\infty}\left[\mathcal{H}_\phi(z, a_+) - \mathcal{H}_\phi(z, a_-)\right]e^{j\eta z}dz, \quad (16)$$

$$\mathcal{I}^-(\eta, a) = \frac{1}{(2\pi)^{1/2}} \int_{-\infty}^{0} [\mathcal{H}_\phi(z, a_+) - \mathcal{H}_\phi(z, a_-)] e^{j\eta z} dz, \quad (17)$$

$$V^+(\eta, a) = \frac{-1}{(2\pi)^{1/2}} \int_{0}^{+\infty} \mathcal{E}_z(z, a) e^{j\eta z} dz, \quad (18)$$

$$V^-(\eta, a) = \frac{-1}{(2\pi)^{1/2}} \int_{-\infty}^{0} \mathcal{E}_z(z, a) e^{j\eta z} dz. \quad (19)$$

$\mathcal{I}_m(\eta) > -W_d$, where $W_d > 0$, $W_d < |\mathcal{I}_m(K)|$, $W_d < |\mathcal{I}_m[(K^2\epsilon - K_c^2)^{1/2}]|$, and $W_d < |\mathcal{I}_m[K(1+q^2)^{1/2}]|$.[3] On the other hand, $F(\eta)$ is analytic and free of zeros inside a narrow strip $2W_d$ about the real axis of the η plane. Therefore, we can rearrange (20) into a new equation where the left-hand side is analytic for $\mathcal{I}_m(\eta) < W_d$ and the right-hand side is analytic for $\mathcal{I}_m(\eta) > -W_d$ as follows:[1]

$$\frac{\mathcal{I}^-(\eta, a)(K+\eta)^{1/2}[\eta + (K^2\epsilon - K_c^2)^{1/2}]}{[\eta + K(1+q^2)^{1/2}] \exp[\xi^-(\eta)]} - \frac{\omega\epsilon\epsilon_0 a A J_1(K_c a)}{(2\pi)^{1/2} K_c [\eta - (K^2\epsilon - K_c^2)^{1/2}}$$
$$\cdot \left\{ \frac{[\eta + (K^2\epsilon - K_c^2)^{1/2}](K+\eta)^{1/2}}{[\eta + K(1+q^2)^{1/2}] \exp[\xi^-(\eta)]} - \frac{2(K^2\epsilon - K_c^2)^{1/2}[K + (K^2\epsilon - K_c^2)^{1/2}]^{1/2}}{[(K^2\epsilon - K_c^2)^{1/2} + K(1+q^2)^{1/2}] \exp[\xi^-\{(K^2\epsilon - K_c^2)^{1/2}\}]} \right\}$$
$$= \frac{\omega\epsilon_0(\epsilon+1) a [\eta - K(1+q^2)^{1/2}] V^+(\eta, a)}{[\eta - (K^2\epsilon - K_c^2)^{1/2}](K-\eta)^{1/2} \exp[\xi^+(\eta)]}$$
$$+ \frac{2\omega\epsilon\epsilon_0 a J_1(K_c a)(K^2\epsilon - K_c^2)^{1/2}[K + (K^2\epsilon - K_c^2)^{1/2}]^{1/2}}{(2\pi)^{1/2} K_c [\eta - (K^2\epsilon - K_c^2)^{1/2}][(K^2\epsilon - K_c^2)^{1/2} + K(1+q^2)^{1/2}] \exp[\xi^-\{K^2\epsilon - K_c^2)^{1/2}\}]} \quad (22)$$

Then the boundary conditions 1), 2), and 3) give[1]

$$\mathcal{I}^-(\eta, a) = \frac{\omega\epsilon_0(\epsilon+1) a [\eta^2 - K^2(1+q^2)]}{\Lambda_a[\eta^2 - (K^2\epsilon - K_c^2)]} F_{(\eta)} V^+(\eta, a) + \frac{\omega\epsilon_0{}^\epsilon a A J_1(K_c a)}{(2\pi)^{1/2} K_c [\eta - (K^2\epsilon - K_c^2)^{1/2}]} \quad (20)$$

where

$$F_{(\eta)} = \frac{j\Lambda_a[\eta^2 - (K^2\epsilon - K_c^2)]}{\left(1 + \frac{1}{\epsilon}\right)[\eta^2 + K^2(1+q^2)]} \cdot \frac{J_1(\Lambda_d a) - \frac{\Lambda_d}{\epsilon\Lambda_a} \frac{H_1^{(2)}(\Lambda_a a)}{H_0^{2}(\Lambda_a a)} J_0(\Lambda_d a)}{\Lambda_d J_0(\Lambda_d a)}. \quad (21)$$

where

$$\xi^-(\eta) = \frac{-1}{2\pi j} \int_{-\infty + jW^d}^{+\infty + jW_d} \frac{\ln F(\xi)}{\xi - \eta} d\xi, \quad (23)$$

$$\xi^+(\eta) = \frac{-1}{2\pi j} \int_{-\infty - jW^d}^{+\infty - jW_d} \frac{\ln F(\zeta)}{\zeta - \eta} d\zeta. \quad (24)$$

$\xi^-(\eta)$ is analytic for $\mathcal{I}_m(\eta) < W_d$. $\xi^+(\eta)$ is analytic for $\mathcal{I}_m(\eta) > -W_d$.

Both sides of (22) are analytic for $|\mathcal{I}_m(\eta)| < W_d$. Thus, they are the analytic continuation of each other. We can show from their asymptotic behaviors that they must both be identically equal to the constant zero.[1]

Hence,

$$V^+(\eta, a) = -\frac{2\epsilon A J_1(K_c a)(K^2\epsilon - K_c^2)^{1/2}[K + (K^2\epsilon - K_c^2)^{1/2}]^{1/2}(K-\eta)^{1/2} \exp[\xi^+(\eta) - \xi^-\{(K^2\epsilon - K_c^2)^{1/2}\}]}{(2\pi)^{1/2} K_c [(K^2\epsilon - K_c^2)^{1/2} + K(1+q^2)^{1/2}](\epsilon+1)[\eta - K(1+q^2)^{1/2}]}, \quad (25)$$

$$\mathcal{I}^-(\eta, a) = \frac{A\omega\epsilon_0\epsilon a J_1(K_c a)[\eta + K(1+q^2)^{1/2}] \exp[\xi^-(\eta)]}{K_c[\eta^2 - (K^2\epsilon - K_c^2)](2\pi)^{1/2}[\eta - K(1+q^2)^{1/2}]}$$
$$\cdot \left[\frac{(K+\eta)^{1/2}[\eta + (K^2\epsilon - K_c^2)^{1/2}]}{[\eta + K(1+q^2)^{1/2}] \exp[\xi^-(\eta)]} - \frac{2(K^2\epsilon - K_c^2)^{1/2}[K - (K^2\epsilon - K_c^2)^{1/2}]^{1/2}}{[(K^2\epsilon - K_c^2)^{1/2} + K(1+q^2)^{1/2}] \exp[\xi^-\{(K^2\epsilon - K_c^2)^{1/2}\}]} \right]. \quad (26)$$

The Solutions of the Far Zone Fields

In slightly dissipative media, K, $(K^2\epsilon - K_c^2)^{1/2}$, and $K(1+q^2)^{1/2}$ must have a small negative imaginary part to comply with the law of conservation of energy. It follows that $\mathcal{I}^-(\eta, a)$ must be analytic for $\mathcal{I}_m(\eta) < W_d$ and that $\mathcal{I}^+(\eta, a)$ and $V^+(\eta, a)$ must be analytic for

In this section, we have regarded (22) valid for all η. Thus the solutions expressed in (25) and (26) are equal to the solutions of $\mathcal{I}^-(\eta, a)$ and $V^+(\eta, a)$ associated with the physical fields only for $|\mathcal{I}_m(\eta)| < W_d$. Never-

[3] P. M. Morse and H. Feshbach, "Methods of Theoretical Physics," McGraw-Hill Book Co., Inc., New York, N. Y., pp. 453–471; 1953.

theless, these $\mathcal{J}^-(\eta, a)$ and $V^+(\eta, a)$ will yield the correct far zone fields through the inverse Fourier transform, since the inverse transform is performed on the real η axis.

Carrying out the inverse transforms of $\mathcal{E}_z$ and $\mathcal{H}_\phi(z,a_+) - \mathcal{H}_\phi(z, a_-)$ by means of the method of steepest descent and then comparing them with (1) to (4), we obtain:[1]

$$CK_0(Kqa) = (2\pi j)^{1/2} \lim_{\eta \to K(1+q)^{1/2}} [\{\eta - K(1+q^2)^{1/2}\} V^+(\eta, a)] \cdot U\left[\frac{\pi}{4} - \frac{\theta}{2} - \arctan\{(1+q^2)^{1/2} - q\}\right], \tag{27}$$

$$g(\theta) = j\left(\frac{2}{\pi}\right)^{1/2} \csc\theta \frac{V^+(K\cos\theta, a)}{H_0^{(2)}(Ka\sin\theta)}, \tag{28}$$

and

$$BJ_1(K_c a) = \frac{-(2\pi)^{1/2}K_c}{\omega\epsilon_0\epsilon a} \lim_{\eta \to -(K^2\epsilon - K_c^2)^{1/2}} \cdot \{[\eta + (K^2\epsilon - K_c^2)^{1/2}]\mathcal{J}^-(\eta, a)\}. \tag{29}$$

The answers here to the reflected coefficient B and transmitted coefficient C are exact. The answer of $g(\theta)$ is only evaluated to the first term of the asymptotic series, but accuracy of $g(\theta)$ obviously can be readily improved by evaluating more terms of the asymptotic series.

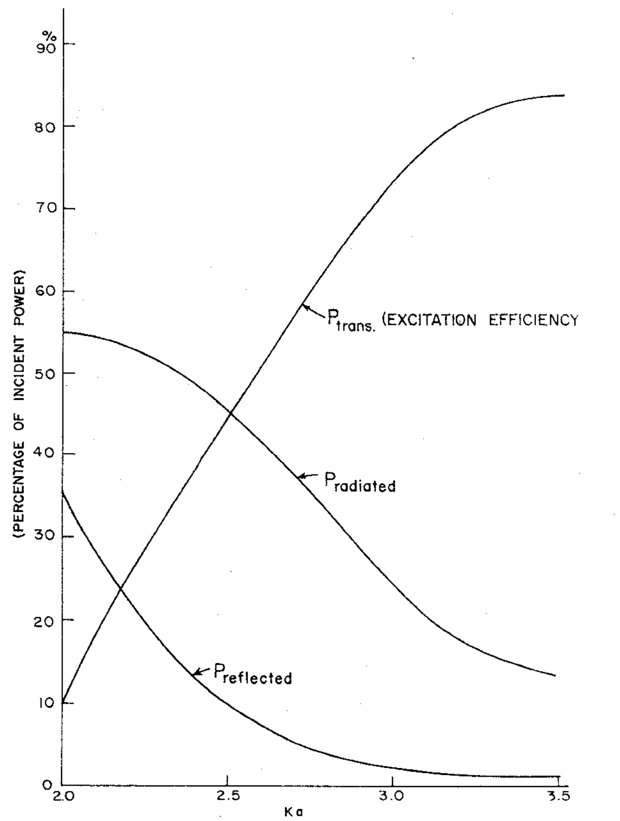

Fig. 2—Plot of the powers radiated, transmitted, and reflected.

The Excitation Efficiency and the Calculation of Power Reflected, Transmitted, and Radiated

Due to the orthogonality of the proper and improper modes, the average power radiated, reflected, and transmitted, can be derived by Poynting's theorem from $g(\theta)$, B, and C, alone. After some tedious manipulations, we obtained the following expression for the reflected power R, transmitted power T (excitation efficiency), and radiation loss L, per unit incident power.[1]

$$T = \frac{4(1+q^2)^{1/2}\left(\epsilon - \frac{K_c^2}{K^2}\right)^{1/2}\left[\left(\epsilon - \frac{K_c^2}{K^2}\right)^{1/2} + 1\right][(1+q^2)^{1/2} - 1]}{q\left[\left(\epsilon - \frac{K_c^2}{K^2}\right)^{1/2} + (1+q^2)^{1/2}\right]^2\left(\epsilon - 1 - \frac{K_c^2}{K^2}\right)^{1/2}} \exp\left[E(1+q^2) - E\left(\epsilon - \frac{K_c^2}{K^2}\right)\right], \tag{30}$$

$$R = \frac{\left[(1+q^2)^{1/2} - \left(\epsilon - \frac{K_c^2}{K^2}\right)^{1/2}\right]^2\left[\left(\epsilon - \frac{K_c^2}{K^2}\right)^{1/2} + 1\right]}{\left[(1+q^2)^{1/2} + \left(\epsilon - \frac{K_c^2}{K^2}\right)^{1/2}\right]^2\left[\left(\epsilon - \frac{K_c^2}{K^2}\right)^{1/2} - 1\right]} \exp\left\{-2E\left(\epsilon - \frac{K_c^2}{K^2}\right)\right\}, \tag{31}$$

$$L = \frac{4\left(\epsilon - \frac{K_c^2}{K^2}\right)^{1/2}\left[\left(\epsilon - \frac{K_c^2}{K^2}\right)^{1/2} + 1\right]\left(p^2 - \frac{K_c^2}{K^2}\right)(1 - \cos\theta)(q^2 + \sin^2\theta)}{\epsilon\pi\left[\left(\epsilon - \frac{K_c^2}{K^2}\right)^{1/2} + (1+q^2)^{1/2}\right]^2\left(\epsilon - 1 - \frac{K_c^2}{K^2}\right)^{1/2}[(1+q^2)^{1/2} - \cos\theta]^2\left(\epsilon - \frac{K_c^2}{K^2} - \cos^2\theta\right)}$$

$$\cdot \frac{(\epsilon - \cos^2\theta)^{1/2} J_0[Ka(\epsilon - \cos^2\theta)^{1/2}] \exp\left[E(\cos^2\theta) - E\left(\epsilon - \frac{K_c^2}{K^2}\right)\right]}{\{4(\epsilon - \cos^2\theta)J_0^2[Ka(\epsilon - \cos^2\theta)^{1/2}] + \pi^2K^2a^2\sin^2\theta M^2(\theta)\}^{1/2}}, \tag{32}$$

for $0 < \theta < \pi$ per unit angle. Where

$$E(\alpha) = \frac{-2\alpha^{1/2}}{\pi K^2a^2} \cdot \int_0^{Ka}\left[\frac{\arctan[W_{(x)}]}{1 - \frac{x^2}{K^2a^2} - \alpha}\right]\frac{x\,dx}{\left(1 - \frac{x^2}{K^2a^2}\right)^{1/2}}, \tag{33}$$

$$W_{(x)} = -\frac{\pi\epsilon x^2 J_1\{[K^2a^2(\epsilon-1)+x^2]^{1/2}\}[J_0^2(x)+N_0^2(x)]}{2[K^2a^2(\epsilon-1)+x^2]^{1/2}J_0\{[K^2a^2(\epsilon-1)+x^2]^{1/2}\}} - \frac{\pi x}{2}[J_1(x)J_0(x) - N_1(x)N_0(x)], \quad (34)$$

$$M(\theta) = J_1[Ka(\epsilon-\cos^2\theta)^{1/2}]\epsilon\sin\theta[J_0^2(\beta)+N_0^2(\beta)] - (\epsilon-\cos^2\theta)^{1/2}J_0[Ka(\epsilon-\cos^2\theta)^{1/2}] \cdot [J_1(\beta)J_0(\beta)+N_1(\beta)N_0(\beta)], \quad (35)$$

$$\beta = Ka\sin\theta. \quad (36)$$

$$\text{Total power radiated} = \int_0^\pi L(\theta)d\theta. \quad (37)$$

Eqs. (30) and (31) are computed numerically on the computer for $\epsilon = 2.49$ for various values of Ka. These numerical results are plotted in Fig. 2. From these results it is clear that

1) The excitation efficiency for $Ka > 3$ is quite high (above 70 per cent). Hence it is an efficient way of excitation. The excitation efficiency does not depend critically upon the precise values of Ka.
2) The larger the normalized cross section of the dielectric rod (Ka) is, the higher is the excitation efficiency.
3) The excitation efficiency curve resembles very closely the efficiency curves obtained in exciting a corrugated surface[4] or a dielectric slab.[5]
4) The excitation efficiency can be improved by flaring out the edge of the metallic waveguide at $z=0$ to accommodate a larger dielectric rod without causing a second mode to propagate inside the metallic waveguide.

[4] A. L. Cullen, "The excitation of plane surface waves," *Proc. IEE*, Monograph No. 93, vol. 101, pt. 4; 1954.

[5] C. M. Angulo and W. S. C. Chang, "On the Excitation of Surface Waves, Part I, II, and III," Div. of Eng., Brown University, Providence, R. I., Scientific Reps. No. AF 1391/3 to AF 1391/5, pp. 67–72; 1956.

An Investigation of the Properties of Germanium Mixer Crystals at Low Temperatures*

L. K. ANDERSON† AND A. HENDRY‡

Summary—**Experimental determinations of the noise temperature ratio, IF resistance, and conversion loss of 1N263 germanium mixer diodes operated in an *X*-band receiver are presented as a function of mixer temperature for the range −196°C to 27°C. No improvement in receiver noise factor was obtained by cooling the mixer to −196°C; however an improvement of 0.3 to 0.6 db was observed by cooling to a temperature in the region −100°C to −50°C. The exact value of the improvement and the optimum temperature depends on the individual crystal, as well as on dc bias and local oscillator drive.**

I. Introduction

It has been suggested,[1] largely on theoretical grounds, that the over-all noise factor of a superheterodyne receiver, employing a germanium crystal mixer, may be improved by cooling the crystal to a temperature substantially below the ambient temperature. The work discussed in this paper was carried out in an effort to verify this prediction, and also to determine how the various crystal parameters, such as IF resistance and noise temperature ratio, vary with temperature in the range from room temperature to the boiling point of nitrogen (about −196°C). The work was carried out at 9375 mc, using type 1N263 germanium diodes.

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† Microwave Lab., Stanford University, Stanford, Calif.; formerly at Natl. Res. Council, Ottawa 2, Ont., Can.

‡ Natl. Res. Council, Ottawa 2, Ont., Can.

[1] G. C. Messenger, "Cooling of microwave crystal mixers and antennas," IRE Trans. on Microwave Theory and Techniques, vol. MTT-5, pp. 62–63; January, 1957.

II. Measurement of Crystal and System Parameters

Fig. 1 is a block diagram of the apparatus, with which the following parameters may be determined: over-all receiver noise factor, IF amplifier noise factor, and the noise temperature ratio, IF resistance, and conversion loss of the crystal mixer. The over-all receiver noise factor and the IF amplifier noise factor are determined by standard methods, *e.g.*, fluorescent lamp waveguide noise source followed by a precision waveguide attenuator for the over-all noise factor, and a temperature limited noise diode with 3-db attenuator in the IF amplifier for the IF noise factor.

The methods used for the measurement of the crystal parameters are largely those described by Torrey and Whitmer.[2]

[2] H. C. Torrey and C. A. Whitmer, "Crystal Rectifiers," M.I.T. Rad. Lab. Ser., McGraw-Hill Book, Co., Inc., New York, N. Y., vol. 15, pp. 223–226; 1948.